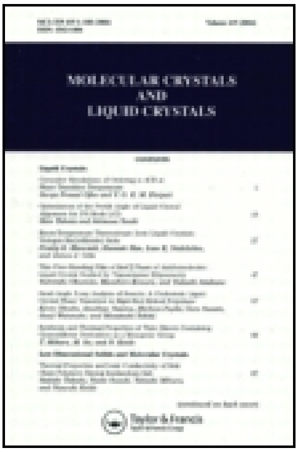

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Theoretical Investigation of Optically Induced Director Oscillations in Nematics

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We report on efforts to develop a theory to understand director oscillations that are generated when a linearly polarized light-wave falls on a homeotropically aligned nematic at a slightly oblique angle. The results of various approaches are compared with each other and to experimental results.

Keywords: Light-Induced Fréedericksz Transition; Chaos; Dynamical phenomena;

INTRODUCTION

The orientational nonlinearity of nematic liquid crystals has been investigated intensively for decades both theoretically and experimentally [1]. Light induced director reorientation occurs when the intensity of the light incident on the LC is sufficiently large so that the orienting effect of the electric field of the light overcomes the elastic forces opposing reorientation in the nematic. The simplest of all these phenomena is the ordinary Light-Induced Fréedericksz Transition (LIFT). Some of the more complex phenomena in this field are the various director oscillations that can occur in certain geometries, for example the director precession induced by circularly and elliptically polarized light [2, 3].

One particular geometry of light-induced director reorientation that has aroused considerable interest is the case of a linearly polarized plane wave incident on a thin cell of homeotropically aligned nematic LC at a small angle α (Fig.1). The direction of polarization is perpendicular to the plane of incidence (ordinary wave). Experiments revealed that as the intensity of the light is increased the director experiences various oscillatory regimes. These oscillations are at first periodic and regular, but at higher intensities various dynamical phenomena and even a transition to chaos can be observed [4, 5]. The experimental data was, however, somewhat difficult to interpret in a number of cases, for example the exact route through which the system becomes chaotic was unclear. This promted continuing interest in trying to develop a theory that would help interpret observations.

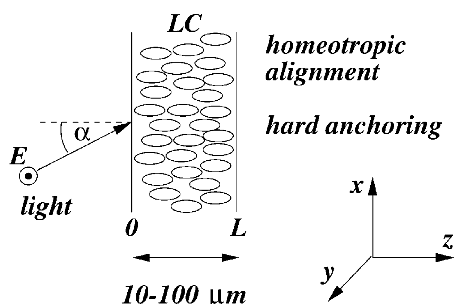

FIGURE 1: *Geometry of the setup: an ordinary wave incident at a slightly oblique angle upon a cell of nematic LC with homeotropic orientation.*

THEORY OF LIGHT-INDUCED DIRECTOR OSCILLATIONS

The basic equations that describe the system are well known, but rather difficult to handle. The equations of motion contain: *i)* the elastic stresses that arise when the orientation of the nematic director is inhomogeneous, *ii)* the torque that the electrical field of the light exerts on the director and *iii)* viscous dissipation. Furthermore, the electric field of the light has to be calculated from the Maxwell equations. Since these contain a dielectric tensor that depends on the director, we have a set of equations that is very difficult to tackle.

To have a hope of obtaining answers, one has to resort to a number of approximations. The most important is the assumption that director components depend only on the z coordinate, i.e. they change only in the direction transverse to the cell. We can then use the plane wave approximation for the light, assuming the diameter of the beam is much larger than the thickness of the layer.

An important simplification results from the hard anchoring condition, as it allows us to write the director as a sum of sine functions: e.g. $n_x = A_m sin(\pi mz/L)$, $n_y = B_m sin(\pi mz/L)$. On observing the structure of the equations, one can see that the elastic damping of m-th mode is proportional to m^2, while the excitation is proportional to the intensity of the light. We can therefore expect that eventually only a finite number (possibly only a few) modes will be important, i.e. the equations will reduce to that of a finite dimensional dynamical system. In this case the oscillations can be described in terms of these few modes whose time evolution is governed by a set of nonlinear ordinary differential equations (ODE-s). The control parameters of the problem are the angle of incidence α and the intensity of the light ρ.

This latter property has been exploited directly in some of the theoretical works, though not all. The first step in developing a theory for the director oscillations is a thorough investigation of the linear stability of the homeotropic state, which can be performed analytically using this mode expansion technique [6, 7]. The results show that the homeotropic state is unstable in a stationary bifurcation for perpendicular incidence of the light and for very small angles. As the angle of incidence is increased, however, the primary bifurcation of the homeotropic state becomes a Hopf bifurcation. The lines of the stationary and the Hopf bifurcations join in a so-called Takens-Bogdanov point, where the basic state possesses a double zero eigenvalue. Figure 2 shows this, calculated for a layer of $50 \mu m$ thickness with material parameters corresponding to the nematic E7 [7] which has been used in most experiments. It can also be seen that the threshold intensity increases sharply with the angle of incidence past the Takens-Bogdanov point. This region proved to be less interesting than the regime of lower angles.

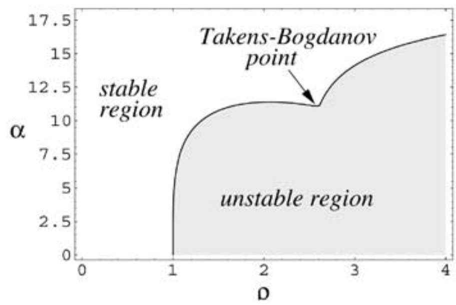

FIGURE 2: *The linear stability of the homeotropic state on the plane of the control parameters* α *(angle of incidence) and* ρ *(intensity of light normalized by the Fréedericksz intensity). The grey area is the unstable region.*

NONLINEAR BEHAVIOUR

To go beyond the linear stability analysis and explore the nonlinear regime, the above mentioned mode expansion has been used to derive a set of three coupled nonlinear ODE-s for the amplitudes A_1, B_1, B_2 [7]. This is the simplest model that can be expected to describe the system, as it contains only three modes (the minimal number required for chaos) and cubic nonlinearities. This model shows that after the initial instability, there are two stable stationary states (fixed points) which correspond to a stationary distorted state of the nematic. There are two states because of the $y \to -y$ symmetry the system possesses - which means that the model equations are invariant under the transformation $S : B_i \to -B_i$. These stationary states also loose stability as the intensity increases. This is a Hopf bifurcation and two limit cycles are born which are mutual images under S. These limit cycles then become homoclinic orbits to the origin (which represents the homeotropic state) at a certain intensity and join to form one single, symmetric limit cycle above that. This is known as a gluing bifurcation and even though a double length limit cycle is created, it is quite different from a period doubling, as the period at the bifurcation point is infinite. Figure 3. shows this sequence of events.

After the first gluing, the symmetric limit cycle undergoes a symmetry-breaking instability in which two asymmetric limit cycles

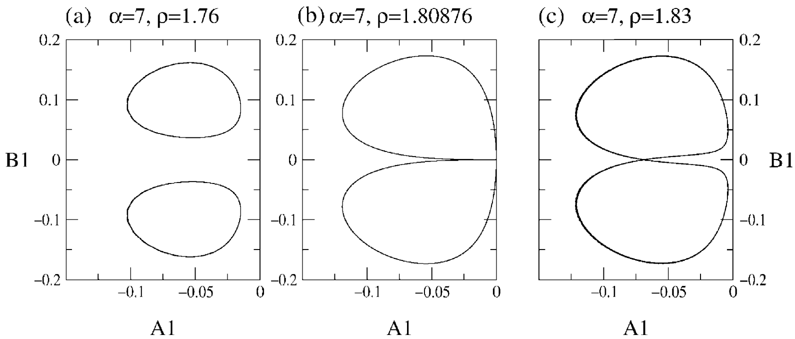

FIGURE 3: *The first gluing bifurcation: the two limit cycles born in the secondary Hopf bifurcation (a) become homoclinic orbits to the origin (b) and glue to form a single, double-length limit cycle (c). The calculation has been performed for a layer of* $50\mu m$ *thickness with material parameters corresponding to E7 [7].*

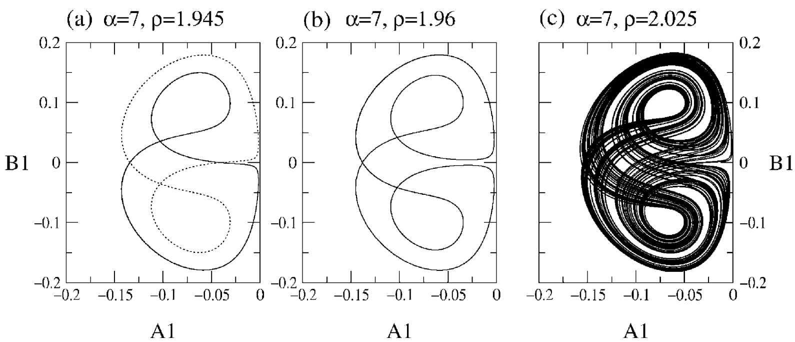

FIGURE 4: *(a) The two asymmetric limit cycles born in the symmetry-breaking bifurcation after the first gluing. (b) Homoclinic orbits in the second gluing bifurcation. (c) Strange attractor after an infinite number of gluing bifurcations.*

are born, mutual images under S (Fig 4.a). Then these two limit cycles glue together to form a single, symmetric, quadrouple length limit cycle in the next gluing bifurcation (Fig 4.b). In what follows, symmetry-breaking and gluing bifurcations follow each other, always doubling the length of the limit cycle until (after an infinite number of bifurcations) a strange attractor is created. This interesting route to chaos was discovered long ago [10], but has never been observed in any real system.

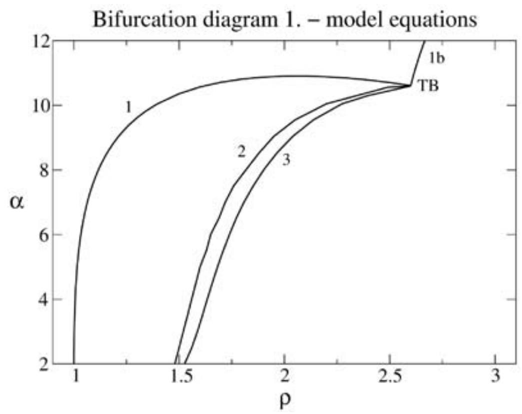

FIGURE 5: *The bifurcation diagram calculated from the model equations. The lines shown are 1) primary bifurcation of the homeotropic state 1b) the primary Hopf bifurcation of the homeotropic state, 2) the secondary Hopf bifurcation of the stationary distorted state and 3) the first gluing bifurcation.*

The bifurcation diagram on the intensity - angle plane as calculated from this model is shown in Figure 5. Both the lines of the secondary Hopf and first gluing bifurcations end in the Takens-Bogdanov point. It is remarkable that the beginning of the sequence of events described so far can be used to interpret experimental observations [5] to a great extent. Periodic limit cycles in the model correspond to regular oscillations in the experiment. The region of the first gluing gives rise to a stochastic regime experimentally, as limit cycles passing very close to each other mean that the system has a chance to "hop" between them due to noise. This stochastic regime in the experiment is again followed by periodic oscillations (the double length limit cycle) and then a second stochastic regime, which can be identified with the situation after the symmetry-breaking instability. Here, once again the system has a chance to make random jumps between limit cycles. The intensities at which each of these bifurcations is supposed to occur, however, do not agree quantitatively with the exerimental values.

The model can also be used to gain physical insight into the reasons for the director oscillations at the onset. First we note that at angles above the Takens-Bogdanov point, where the primary instability of the homeotropic state is a Hopf bifurcation, the electric field of the light wave is stongly distorted by even a slight reorientation of the director. If the reorientation is proportional to $sin(\pi z/L)$, the field that develops is such that it excites the second reorientation mode $sin(2\pi z/L)$. Conversely, if the reorientation is proportional to $sin(2\pi z/L)$, the field that develops drives the first reorientation mode. Hence the oscillation can be understood as a competition between the two reorientation modes, that are coupled by the field. For angles bellow the Takens-Bogdanov point, we note that the initial stationary distorted state is such that the angle between the incident wave and the director increases, i.e. if $k_x > 0$, then $n_x < 0$. As the intensity increases, so does the director reorientation up to the point where stability is lost in a Hopf bifurcation. So the mechanism leading to the oscillations can be very similar to the one described for the homeotropic state. As for the gluing bifurcations or chaotic regime, it is very difficult to give such a simple physical picture, as the motion is much more complex.

To get more accurate values for the threshold intensities for the bifurcations we must resort to a direct simulation of the director equations [8] which allows us to relax most of the approximations used in deriving the model equations. It turns out that while some of the bifurcations described above can also be seen in the simulations, the threshold intensities do indeed become very different. The bifurcation diagram that was calculated from the simulations can be seen on Figure 6. The line of the first bifurcation follows a very different path accross the parameter plane, even though the sequence of events leading to the first gluing is the same as for the model. The quantitative agreement with experiment is somewhat better in this case, but is still far from perfect. While the intensity for the first gluing obtained from the model seems to be too low compared with experiment, the value given by the simulations seems to be too high.

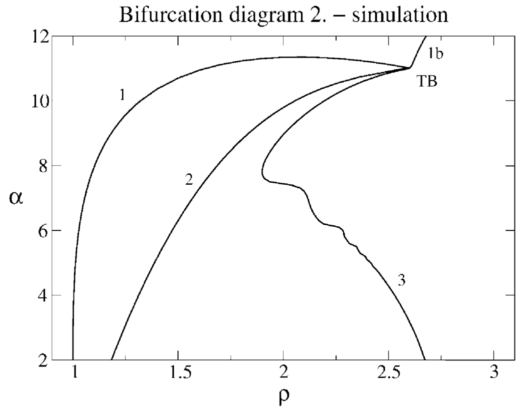

FIGURE 6: *The bifurcation diagram calculated from numerical simulations. The lines shown are the same as on Figure 5.*

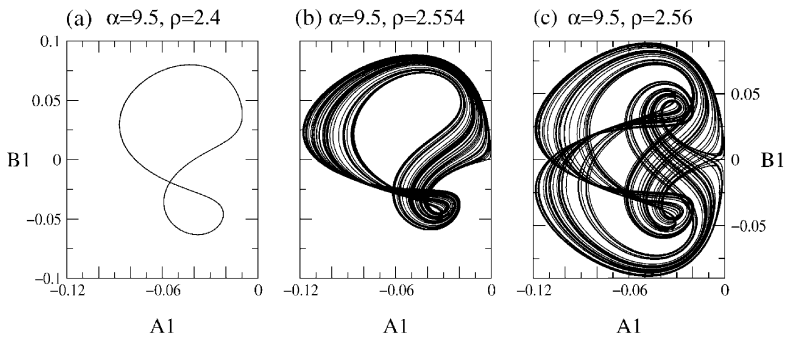

FIGURE 7: *The evolution of the system calulated from the simulations: (a) An asymmetric limit cycle after the symmetry-breaking bifurcation (only one is shown) (b) Asymmetric strange attractor after the perion doubling scenario. (c) Symmetric strange attractor.*

The reason for the discrepancy is not known at the moment.

At higher intensities we find qualitative differences between the predictions of the model equations and that of the simulations. As opposed to the model, which suggests that the system reaches chaos through a sequence of gluing bifurcations, the simulation suggests that after the symmetry-breaking bifurcation creates two asymmetric limit cycles (Fig.7(a)), these go through a period-doubling scenario (Fig.7(b)). The asymmetric strange attractors then join in a "gluing" bifurcation to form a symmetric strange attractor very similar to that appearing in the model (Fig.7(c)).

As a conclusion, we can say that the nonlinear behaviour described by the model and the simulation have been very helpful in interpreting experimental observations for moderate intensity values. A sequence of periodic and stochastic regimes can be explained qualitatively by a gluing bifurcation and a symmetry-breaking bifurcation that follows it. However, nonlinear behaviour and chaos at higher intensities is not yet completely understood.

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References

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[1] N. V. Tabiryan, A. V. Sukhov and B. Ya. Zel'dovich, Mol. Cryst. Liquid Cryst. **136**, 1-140 (1985); F. Simoni, Nonlinear optical properties of liquid crystals World Scientific, Singapore 1997

[2] E. Santamato et al., Phys. Rev. Lett. **57**, 2423 (1986); L. Marrucci et al., Phy. Rev. A **46**, 4859 (1992)

[3] E. Santamato et al., Phys. Rev. Lett. **64**, 1377 (1990)

[4] G. Cipparrone et al., Phys. Rev. E **47**, 3741 (1993);

[5] G. Russo, V. Carbone and G. Cipparrone, Phys. Rev. E **62**, 5036 (2000); V. Carbone, G.Cipparroneand and G. Russo, Phys. Rev. E **63**, 051701 (2001)

[6] N. V. Tabiryan et al., Opt. Commun. **154**, 70 (1998)

[7] G. Demeter and L. Kramer, Phys. Rev. Lett. **83**, 4744 (1999); G. Demeter, Phys. Rev. E **61**,6678 (2000)

[8] G. Demeter and L. Kramer, Phys. Rev. E **64**, 020701 (2001)

[9] B. L. Winkler et al., Phys. Rev. A **43**, 1940 (1992)

[10] A. Arneodo, P. Coullet and C. Tresser, Physics Letters **81A**, 197 (1981); Y. Kuramoto and S. Koga, Physics Letters **92A**, 1 (1982)